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Research Article

Orbital And Spin Effects for Learning the Classification of Magnetic Materials in Engineering Electromagnetics

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Abstract

This study presents a teaching methodology to propose an example to understand the quantum physics model based on classical physics and to build two arrays for studying conceptual orbital and spin effects. The elements of the two conceptual arrays are associated with parameters such as magnetization intensity and relative permeability of the magnetic materials. The results achieve a superior form for explaining the phenomenon of magnetization. Therefore, this study delivered some suggestions to the curriculum design of high school, undergraduate, graduate levels, and textbooks. Compared with courses related to AI which become high school classes in many countries, the importance of magnetic informatics (MI) might be neglected; and that ought to be remedied.

Index Terms: Classification, Ferroelectrics, Magnetic materials, Orbital, Spin

Introduction

Magnetics is usually harder to learn than electrostatics. This statement is true not only for students but also for most engineers and scientists [1,2]. The reason is that most properties in electrostatics are linear and use inner products, but most properties in magnetics are curves and employ outer products. Therefore, electrostatics is intuitive, but magnetics is abstract. Another possible reason is associated with the curriculum design regarding instruction on electromagnetics. Most textbooks and courses related to electromagnetics are arranged to begin with electrostatics, which includes the basic and easy topics that most educators consider readily teachable. Typically, the students who are taking the course do not absorb electrostatics completely, and then they suffer as they fail to learn magnetics. Thus, students must prepare more before learning magnetics. In fact, most educators might agree that electromagnetics has a steep learning curve.

The magnetic properties of materials are widely employed in areas such as medicine and industry. The topic is crucial, but some educators think it is part of materials science. In most textbooks, lessons covering magnetic materials in units on electromagnetics, especially in engineering electromagnetics, might just present a simple overview. In materials science, magnetic properties are presented in terms of basic quantum physics [3,4]. To improve students' learning of electromagnetics, this paper proposes a simple model to explain the classification of magnetic materials in engineering electromagnetics classes. Some books illustrated the formation of domain walls [5,6]. The illustrations indicate 4 types of domain walls whose detailed properties, and dependence on applied magnetic fields are too complex to describe in the textbooks of undergraduate level. Furthermore, the crystal structure did not explain the domains well because are many exceptions in some crystallographic types [5]. The magnetic behavior of a material is used as the basis to classify materials into the categories of diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic, ferromagnetic, or superparamagnetic. It is caused by the interaction of the magnetic dipole moments of the atoms with an external magnetic field.

Each atom contains various component moments. The combination of the moments yields the magnetic characteristics of the material (which can be classified by relevant magnetic behaviors [7-9]). Applications of magnetic materials include saving energy, energy storage, and high-tech electronic devices. From the viewpoints of education and science development, the topic might be examined in the modern curriculums of electrical engineering, electronics, computer science, and other relevant departments aiming to address challenges such as global warming. Some modern computation devices, such as hyper-controlled-not gate [10], path-polarization hyperentanglement on a chip [11], and Raman quantum memory [12] are based on domain knowledge of magnetics and polarization. *Li, et al.,* [13] studied the computation results for a voltage-induced 180° perpendicular magnetization switching in a single magnetic nanoelement. They demonstrated a Ni ellipse, in which the long axis was 60nm, the short axis was 30nm, and the thickness was 7.5nm on a substrate of piezoelectric ceramics are lead zirconate titanate (PZT). The strain pulse decided the directions of the magnetization. The results proved that single-bit Ni elements can be controlled to serve as a 180°-switching device.

In addition, *Bromberg, et al.,* indicated that magnetic logic has recently become an attractive candidate for future electronics. They described the spintronic device, mCell, at first, then developped mLogic with gain sufficient to drive fanout independent of CMOS [14]. Some recent patents such as magnetic logic devices proposed by *Shum, et al.,* [15], and integrated magnetic random-access memory with logic device announced by *Kondo, et al.,* [16] pointed out the importance of magnetic aterials in the future learning of physics, electrical and electronic engineering. To sum up, the importance of magnetic materials is increasing; it may become the mainstream of quantum computation. Currently, artificial intelligence (AI) plays an important role in information and computer engineering; AI is taught in middle and high schools in many countries. If quantum computation matures, the basic logic gate design of quantum computation might be a new star of material science soon. Therefore, educators must analyze and compare relevant learning materials.

This article is structured into six sections. The first section is the introduction to magnetic materials and a literature review. The second section describes a general description for magnetic materials based on classical and quantum physics, respectively. The third section shows a diamagnetic case as an example. The fourth section presents the types of magnetic behaviors and introduces the conceptual arrays. The fifth section discusses the challenges and advantages of the array form we propose in this article. Finally, the conclusion and directions for future work are discussed in the sixth section.

General Description for Magnetic Materials

Primarily, magnetic behaviors in materials are based on two effects: orbital and spin. Therefore, this study focused on two methods to explain the behavior in materials: quantum physics and classical physics. The methods can be described as follows.

Classical Physics

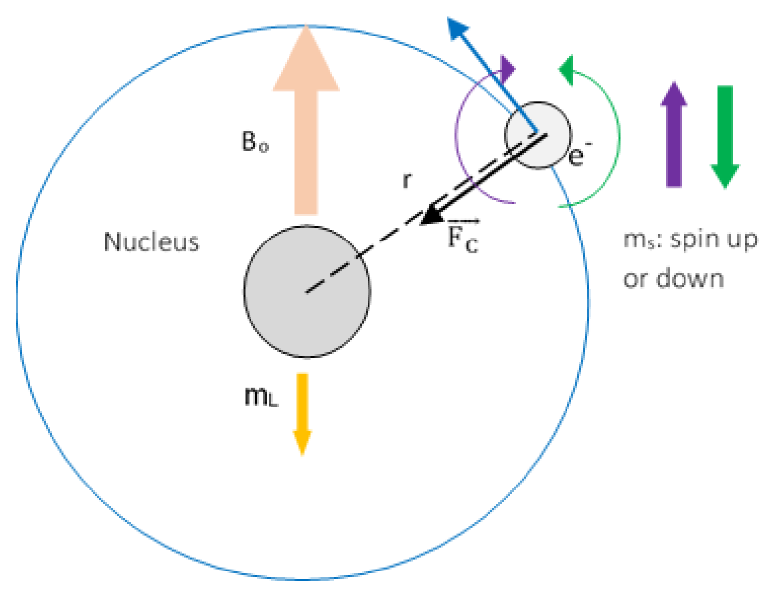

Figure 1: An orbital magnetic moment (m_L), and a spin magnetic moment (m_S) in a simple atomic model.

In Figure 1, an extremely small current loop with current can be described as

$$I = \frac{e}{T} = \frac{-ev}{2\pi r} \qquad (1)$$

where v is the velocity of the electron (which is moving in a circular orbit of radius r). Therefore, mL can be written as

$$m_L = IA = -\frac{-evr}{2} = -(\frac{e}{2m})L \qquad (2)$$

where L is classical angular momentum:

$$L = mvr \qquad (3)$$

For the spin in the atomic model in Figure 1, suppose the electron is a classical solid sphere with radius (re):

$$r_e = \frac{-e^2}{4\pi\varepsilon_o mc^2} \qquad (4)$$

where is the vacuum permittivity, and c is the vacuum light speed.

$$mv = p \approx \Delta p = \frac{h}{r_e} = \frac{h^2 mc^2}{e^2} \approx 137mc \qquad (5)$$

That is,

$$v \approx 137c \qquad \text{conflict} \qquad (6)$$

This conflicts with Eq. (6). It is impossible that the velocity of electron is larger than c, and thus, this model does not correspond [17,18].

Quantum Physics

Figure 1 is a simple atomic model. The magnitudes of the corresponding orbital magnetic moment (m_L) and spin magnetic moment (m_S) can be written as follows [19]:

$$m_L = g_l(\frac{-e}{2m})L \qquad (7)$$

$$m_S = g_S(\frac{-e}{2m})S \qquad (8)$$

where g_l and g_s are Lander g-factors, e is the magnitude of electron charge, m is the mass of an electron, L is the angular momentum operator of the orbital, and S is the angular momentum operator of the spin in quantum physics. In pure-sample cases, g_l = 1 and g_s = 2; the Lander g-factors are more complicated if L and S are coupling with each other.

Based on the circular movement in classical physics, mS can be written as follows [18]:

$$m_S = (\frac{-e}{2m})S \qquad (9)$$

where mS is the same to the unit orbital magnetic moment; g_S = 2 in Eq. (1) from the viewpoint of quantum physics. We must indicate that, ideally, the direction of mS is up if the microspin current is clockwise (purple arrow) but down if the microspin is counterclockwise (green arrow), where the direction of current is opposite to the definition of electron spin. In brief, we define the antiferromagnetic material as a material in which the directions of mS are opposite.

Case as an Example: Diamagnetic

A diamagnetic material case was presented as a result in the methods of quantum and the classical physics. In Figure 1, an external magnetic field $\vec{B}$ was applied. If the system has no spin, then the case can be analyzed through the following two methods.

Classical Physics

Consider Figure 1, $\vec{B}$ exerts a magnetic force on the electron ($\vec{F_B}$). Therefore, the electrical force ($\vec{F_e}$) provides the centripetal force ($\vec{F_c}$) between nucleus and electron.

The total force ($\vec{F_t}$)on the electron in Figure 1 is the summation of $\vec{F_e}$ and $\vec{F_B}$. Therefore,

$$\vec{F_t} = \vec{F_c} = \vec{F_e} + \vec{F_B} \qquad (10)$$

Without any magnetic field,

$$\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} = \mathrm{m}\frac{v^2}{r} \qquad (11)$$

Then (10) can be rewritten as

$$\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} + e\tilde{v}B = \mathrm{m}\frac{\tilde{v}^2}{r} \qquad (12)$$

where $\tilde{v}$ is the new velocity after giving the B. Then,

$$e\tilde{v}B = \frac{m}{r}(\tilde{v}^2 - v^2) = \frac{m}{r}(\tilde{v} + v)(\tilde{v} - v) \qquad (13)$$

Suppose the change Δv is small, then

$$\Delta v = \tilde{v} - v \qquad (14)$$

Take Eq. (14) into Eq. (13), then

$$\Delta v = \frac{erB}{2m} \qquad (15)$$

The electron is accelerated by B. Accordingly, the orbital dipole moment is

$$m_L = -\frac{1}{2} evr\hat{z} \qquad (16)$$

Therefore, the change in the dipole moment can be written as

$$\Delta m_L = -\frac{1}{2}\mathrm{e}(\Delta v)r\hat{z} = -\frac{e^2r^2}{4m}\mathrm{B} \qquad (17)$$

Quantum Physics

The above result can be deduced from quantum physics. According to quantum physics, we have a Hamiltonian:

$$H = \sum_{i=1}^{n} \frac{1}{2m}[P_i + e\vec{A}(\vec{r_i})]^2 + \mathrm{V}(\vec{r_1}, \vec{r_2}, \vec{r_3}, \cdots, \vec{r_n}) \qquad (18)$$

Because of

$$B = \nabla \times \vec{A} \qquad (19)$$

Assume B is along the z-axis, then the vector potential is presented as

$$\vec{A} = \frac{1}{2}(-B_z\mathrm{y}, B_z x, 0) \qquad (20)$$

Choose the Coulomb gauge:

$$\nabla \bullet \vec{A} = 0 \qquad (21)$$

The Hamiltonian is written as

$$H = H_o + \frac{eB_z}{2m}\mathrm{L}_z + \frac{e^2B_z^2}{8m}(\mathrm{x}_i^2 + \mathrm{y}_i^2) \qquad (22)$$

where Ho is zero-order Hamiltonian without a magnetic field:

$$H_o = -\sum_i \frac{\hbar^2}{2m}\nabla_i^2 + \mathrm{V} \qquad (23)$$

The appendix presents the derivation of Eq. (23) in detail. The ground state wave function is represented by two quantum numbers L, M_L; one must calculate the first-order perturbation. Therefore,

$$\Delta E = \frac{eB_m}{2m} < L, M_L \left| L_z + \frac{eB_z}{4}\sum_{i=1}^{n}(x_i^2 + y_i^2) \right| L, M_L > \qquad (24)$$

According to thermodynamics, we have

$$M_Z = -\frac{\partial(\Delta E)}{\partial B_Z} \qquad (25)$$

Therefore, the first term of M_z must be

$$M_Z^{(1)} = -M_L \mu_B \qquad (26)$$

where μ_B is Bohr magneton, which does not depend on any magnetic field. The second term of Mz is written as

$$M_Z^{(2)} = -\left(\frac{e^2}{4m}\right)\mathrm{B}_z < \mathrm{L}, \mathrm{M}_L \left| \sum_{i=1}^{n}(x_i^2 + y_i^2) \right| \mathrm{L}, \mathrm{M}_L > \qquad (27)$$

It depends on the magnetic field, and the minus sign indicates the diamagnetic effect [18-20]. In fact, Eq. (17) is a special case of Eq. (27). In a simple atomic model, r2 in Eq. (17) is equal to x2+y2 in Eq. (27) which represented a quantum form of a wave equation to indicate the main disturbance. The special case simplifies the understanding of Eq. (27) which is a general form of any atoms. It is easy for students to read the meaning of Eq. (17) through the understanding of Eq. (17). The understanding of Eq. (17) aided the recognition of the general form in Eq. (27). The disturbances influence the classification of the magnetic materials.

Types of Magnetic Behaviours

The total magnetic effect can be regarded as combined effects from m_L and m_S. Therefore, based on effect, the magnetic behaviors of materials can be categorized into paramagnetic, diamagnetic, ferromagnetic, and antiferromagnetic. The diamagnetic effect is discussed in the above section. We focus on paramagnetic, diamagnetic, ferromagnetic, and antiferromagnetic in this section. Roughly speaking, the effects mainly come from the spin. Based on the aforementioned application of an external magnetic field, those domains (which have moments in the direction of the applied field) increase the influences of their neighbors, and the internal magnetic field enhances much more than the external field. Then, after the external field has been removed, a completely random domain alignment is not usually attained and a resident (or remnant dipole field) remains in the macroscopic structure. Consequently, the antiferromagnetic material has no net magnetic dipole moment, the paramagnetic material has a weak total permanent magnetic dipole moment, and the ferromagnetic material has a strong total permanent magnetic dipole moment. Figure 2 presents the magnetic behaviors [8,17].

Regarding Figure 2, we define O as the distribution of orbital effect and S as the distribution of spin-up or spin-down responding to the atom positions:

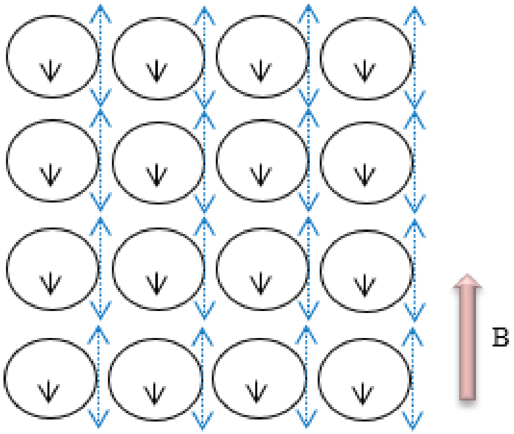

Figure 2: Illustration of magnetic behaviors. ↓: Orbital effect (always opposite to Bo); ↕ : Spin effect (spin up or down depend on possibility).

$$O = \begin{pmatrix} o_{11} & o_{12} & o_{13} & o_{14} \\ o_{21} & o_{22} & o_{23} & o_{24} \\ o_{31} & o_{32} & o_{33} & o_{34} \\ o_{41} & o_{42} & o_{43} & o_{44} \end{pmatrix} \quad (28)$$

and

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix} \quad (29)$$

In addition, the magnetic flux density in the material (B) is

$$B = B_0 + B_m = \mu_0(H_0 + M) = \mu_0(1 + \chi_m)H_0 \quad (30)$$

where Ho is the external magnetic field, and μ_0 is permeability of free space. Therefore, $B_o = \mu_0 H$, M is magnetization intensity (which means the density of magnetic dipole moment), and χ_m is the magnetizability [21-24]. Let

$$\mu_r = 1 + \chi_m \quad (31)$$

where μ_r is the relative permeability of a material; then we have

$$B = \mu_0 \mu_r H_0 \quad (32)$$

Antimagnetic Material

For an antimagnetic material, the orbital and spin effects are close to zero. Consequently, an extremely simplified case for the presentation of O and S can be written as

$$O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (33)$$

where 0 means no orbital effects. Furthermore,

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (34)$$

where 0 represents that the spin effects are canceled by the neighbors. Therefore, it is easy to find that B approaches 0 and M is close to null from (33) and (34). Some elements, such as Bi and Pb, are antimagnetic [25].

Diamagnetic Material

For a diamagnetic material, the orbital effects are dominant. An extremely simplified case for the presentation of O and S is

$$O = \begin{bmatrix} -\delta & -\delta & -\delta & -\delta \\ -\delta & -\delta & -\delta & -\delta \\ -\delta & -\delta & -\delta & -\delta \\ -\delta & -\delta & -\delta & -\delta \end{bmatrix} \quad (35)$$

where -δ means a small orbital effect such that the direction of the orbital effect is opposite to that of Bo. In addition,

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (36)$$

The spin effects are also canceled by the neighbors. Therefore, it is easy to find that $B<B_0$ from Eq. (35) and Eq. (36). Some typical diamagnetic materials are Cu, Au, and Hg [26]

Paramagnetic Material

For a paramagnetic material, the spin effects are dominant. An extremely case for simplified presentation of O and S were written as,

$$O=\begin{bmatrix}-\delta & -\delta & -\delta & -\delta\\ -\delta & -\delta & -\delta & -\delta\\ -\delta & -\delta & -\delta & -\delta\\ -\delta & -\delta & -\delta & -\delta\end{bmatrix} \quad (37)$$

And

$$S=\begin{bmatrix}-\frac{1}{2} & 0 & 0 & -\frac{1}{2}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ +\frac{1}{2} & 0 & 0 & +\frac{1}{2}\end{bmatrix} \quad (38)$$

where (s12, s13), (s22, s23), (s32, s33), (s42, s43), (s21, s31), and (s24, s34); their spin effects are also canceled by the neighbors. Concurrently, s11 s41 s14 and s44 are induced to magnetic dipole moments occasionally. The bold +1/2 and −1/2 indicate the formation of magnetic dipoles. Therefore, it is easy to find that $B>B_0$, $\mu_r>1$, $\chi_m>0$ from Eq. (35) and Eq. (36). A typical paramagnetic material is Al [27].

Ferromagnetic Material

For a ferromagnetic material, the spin effects are dominant. Thus, the descriptions of O and S can be written as

$$O=\begin{bmatrix}-\delta & -\delta & -\delta & -\delta\\ -\delta & -\delta & -\delta & -\delta\\ -\delta & -\delta & -\delta & -\delta\\ -\delta & -\delta & -\delta & -\delta\end{bmatrix} \quad (39)$$

and

$$S=\begin{bmatrix}-\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ +\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2}\end{bmatrix} \quad (40)$$

where raw 2 and raw 3 produce the domain walls. Concurrently, raw 1 and raw 4 are induced to be permanent magnetic dipole moments. Therefore, it is easy to find that $B >> B_0$, $\mu_r \gg 1$, $\chi_m \gg 1$ from Eq. (39) and Eq. (40). Some typical ferromagnetic materials are Fe, Co, and Ni [28].

Discussion

Equation (5) is conflict. However, Eq. (5) was corrected to be Eq. (17) by the derivations of the equations which based on the theories of classical physics. Eq. (17) provides a successful explanation of the change in the dipole moment. It is much easier for students to learn it in the knowledge of classical physics, especially for the students in the engineering colleges. However, based on Einstein's theory of general relativity, m should be increased. It is concerned with the theory of general relativity in modern physics. Therefore, we induced the exact form of Eq. (27).

The derivations of Eq. (17) and Eq. (27) are in the same way. Therefore, Eq. (27) is a typical example to understand the quantum physics model based on classical physics. It certainly brings the concise and convinient idea to enter the world of modern physics for the undergratuates of engineering colleges. This is an important contribution of this study to encourage the lecturers to study the teaching methods to make students easy to learn the future world of modern physics. The proposed arrays O and S enhance the connections need to understand the illustration in Figure 2; this enables effective presentation of the magnetic effects of orbital and spin. The learning of arrays usually begins during high school for most students. Therefore, it is possible to introduce arrays to some talented high school students. A college student can mentally connect our proposed arrays O, S, and M with Eq. (33) to Eq. (40) to improve the learning of Eq.s. (30), (31), and (32) because the values of the elements in arrays O and S lead to a clear image of the magnetization. Consequently, the proposed arrays O and S achieve a particularly simple and clear presentation of material classifications and magnetic behaviors, specifically those of antimagnetic, diamagnetic, paramagnetic, and ferromagnetic materials.

Kittel illustrateed the formation of domains [6]. Reviewing this textbook, the conceptual arrays of Eq. (40) can be illustrated as Figure 3. The structure of the Bloch wall separating domains displays conceptually. This is the mechanism of the ferromagnetic material which is the basis of modern technologyies such as eletric vehicles (EV), motors, and power generators. Therefore, the conceptual arrays function as the llustrations with much clear and reasonable presentation based on the orbital and spin effects. This is one of the contributions of this study, i.e., to elevate the understandings using the simple array forms whose performance are as well as the inllustration forms. Obviously, the illustration of Kittel's study strongly supports the descriptions of O and S.

Figure 3: Equation (40) illustrates the structure of the Bloch wall separating domains in Kittel's book [6] conceptually.

It is notable that the magnetic logic devices is a potiential field of the semiconductor industry. *Luo, et al.,* [29] developped the magnetic domain wall logic devices driven by current pulses. Because of the advantages of nonvolatile data retention, near-zero leakage, and scalability of spin-based logic architectures, the technology roadmap beyond CMOS logic is very possible to be new star in this industry [30-35]. Therefore, we suggest to the design of the curiculumn as follows,

1. In high school level, the basic concept of magnetic materials might be introduced in the courses of physics.
2. In undergraduate and graduate levels, the organization of a course program which includes solid state physics, microelectronics, digital electronics, and spin electronics is necessary for the future demands of the industry.
3. In the textbooks related to electromagnetics, some basic examples of the magnetic logic gates might add in the chapter of magnetic materials. We believe that the new examples will enhance the interest of students' learning.

The classification of magnetic materials is not the core issue in electromagnetics. However, advanced semiconductor technology is developing logic devices with magnetic materials [12,36]. Educators might keep an eye on the trend of applications of magnetic materials to motivate students with the prospect of learning practical science. Each department designs its own curriculum to suit its own particular objectives. The curriculum designs for fundamental sciences (e.g., physics and chemistry) and those for applied engineering (e.g., electrical engineering, electronic engineering, and computer science) are different. However, quantum physics is a core course in departments of physics, but it might be an elective course in departments of engineering. Therefore, the proposed method provides the content of a brief lecture to explain magnetization and magnetic behaviors in courses of an engineering college (e.g., engineering electromagnetics).

Conclusion

Our study employed a simple classical equation in a simple atomic model (which was based on classical physics) to explain a general form in quantum physics. Because quantum physics is typically not required for engineering students, our proposed method is very helpful to aid engineering students in learning about magnetic materials in the course of engineering electromagnetics or applied electromagnetics without any prerequisite knowledge of quantum physics. The elements of O and S associate with parameters such as M, χ_m, and μ_r of the magnetic materials to explain the equations related to magnetization intuitively. The domain walls disclose in the arrays to elevate the understanding of the property not only in quality but also in quantity. The results yield a better presentation for explaining the phenomena of magnetization.

The findings proved that the method of quantum physics for students is based on quantum dynamics, but the method of classical physics for students who have learned fundaments of physics is a special case of the formulas derived from quantum physics. Therefore, the method of classical physics is beneficial for some students who have merely learned the fundamentals of physics in departments of electrical and computer engineering. The technology development of digital computers and microprocessors is in the trend of lower power consumption, higher density of logic gates, and higher computation speed to support the mobility and complex computation for tablets or smartphones. Exactly, magnetics and magnetic materials support the potential knowledge for the advanced devices. Consequently, compared with the courses related to AI which becomes high school classes in many countries, the importance of the magnetic informatics (MI) might be neglected; this study corrects that underperformance.

Appendix

The derivation of (23) was presented as follows,

$$\vec{A} = \frac{1}{2}(-B_z y, B_z x, 0) \quad \text{(A1)}$$

$$P_i = \nabla_i \quad \text{(A2)}$$

$$H = \sum_{i=1}^{n}\left[-\frac{\hbar}{2m}\nabla_i^2 + \frac{e\hbar}{mi}A.\nabla_i + \frac{e^2}{2m}A^2\right] + V \quad \text{(A3)}$$

$$H=\sum\nolimits_{i=1}^{n}\left\{-\frac{\hbar}{2m}\nabla_i^2+\frac{eB_z}{2m}\left[\frac{\hbar}{i}(x_i\frac{\partial}{\partial y_i}-y_i\frac{\partial}{\partial x_i})\right]+\frac{e^2B_z{}^2}{8m}(x_i^2+y_i^2)\right\}+V \quad \text{(A4)}$$

$$H=H_0+\frac{eB_z}{2m}\sum\nolimits_{i=1}^{n}\left[\frac{\hbar}{i}(x_i\frac{\partial}{\partial y_i}-y_i\frac{\partial}{\partial x_i})\right]+\frac{e^2B_z{}^2}{8m}\sum\nolimits_{i=1}^{n}(x_i^2+y_i^2)+V \quad \text{(A5)}$$

i.e. $$H=H_0+\frac{eB_z}{2m}L_z+\frac{e^2B_z{}^2}{8m}\sum\nolimits_{i=1}^{n}(x_i^2+y_i^2) \quad \text{(A6)}$$

$$H_0=-\sum\nolimits_{i=1}^{n}\frac{\hbar}{2m}\nabla_i^2+V \quad \text{(A7)}$$

The Eq. (A6) and Eq. (A7) are Eq. (22) and Eq. (23), respectively.

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