

Review Article

Copyright© Jorge Garza Ulloa

Electromagnetic Field Toroidal Equations Applied to Magnetic Confinement Fusion as Tokamak

Jorge Garza Ulloa*

Research Consulting Services, University of Texas El Paso, USA

*Corresponding author: Jorge Garza Ulloa, Research Consulting Services, University of Texas El Paso, USA.

To Cite This Article: Jorge Garza Ulloa*. Electromagnetic Field Toroidal Equations Applied to Magnetic Confinement Fusion as Tokamak. *Am J Biomed Sci & Res.* 2025 27(5) AJBSR.MS.ID.003602, DOI: [10.34297/AJBSR.2025.27.003602](https://doi.org/10.34297/AJBSR.2025.27.003602)

Received: 📅 July 02, 2025; **Published:** 📅 July 11, 2025

Abstract

This paper is a part of *APPLIED ELECTROMAGNETIC FIELDS PROJECT*, where the main goal is the design of electromagnetics field devices for special applications. Making focus in the toroidal coil shape that have the ability to concentrate its magnetic fields in its axial center, that can be used as confinement fusion systems. In this research paper the objectives are:

- Introduction to electromagnetic fields and equations. These equations will be used in this paper and the rest of my *APPLIED ELECTROMAGNETIC FIELDS PROJECT*.
- Study the four Maxwell equations that can be applied in different confinement fusion systems to handle the plasma and their extreme high temperature.
- Deductions of the toroidal magnetic equations to be applied in many electromagnetics applications from the typical transformer to the Magnetic Confinement System, and
- Explaining different Thermonuclear Confinement Systems as: Gravitational, Inertial, Electrostatic, Tokamaks (special emphasis), Stellarators and Magnetic mirror.

Learning the electromagnetic concepts and applying them in electromagnetic methods of generate energy in the near future, that can be applied in the design of many other useful electromagnetic devices (Figure 1).

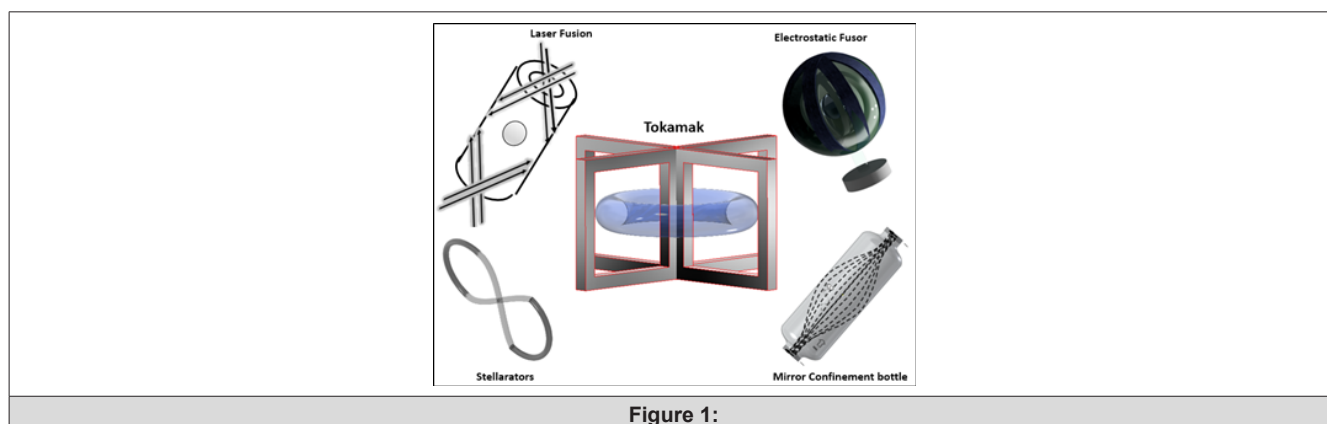


Figure 1:

Highlights: Maxwell equations, Confined Gravitational, Inertial confinement, Electrostatic confinement, Tokamaks, Stellarators, Magnetic mirror confinement

Introduction to Electromagnetic Fields Concepts and Equations

According with the Newton's law of gravity for gravitational force that state that every electric particle attracts every other elec-

tric particle in the universe with a force which is directly proportional to the products of their masses and inversely proportional to the square of the distance between their centers as stated in eq. 1 and shown in Figure 2A.

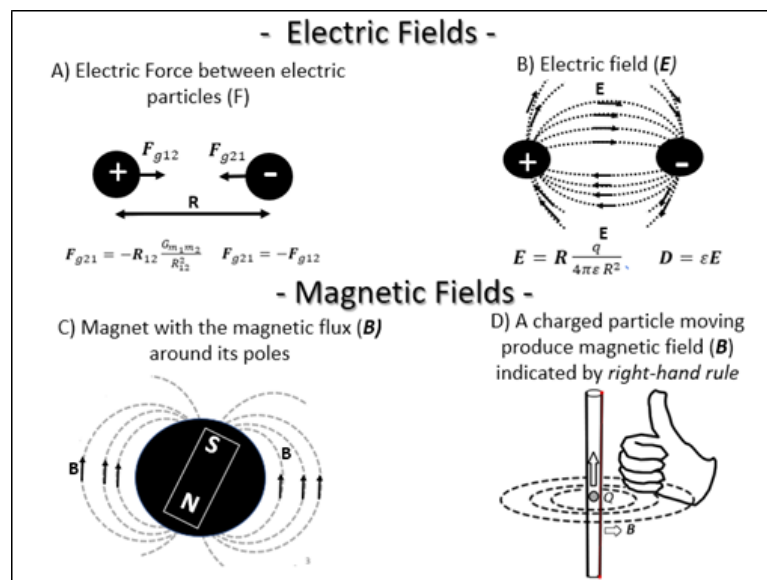


Figure 2: Electric and magnetic fields.

A) Electric force vector, B) Electric field vector (E), C) Magnetic field vector and D) Magnetic field vector (B) direction.

Eq. 1) Newton's law of gravity for gravitational force

$$F_{g21} = -R_{12} \frac{G_{m1}m2}{R_{12}^2} (N)$$

where: $F_{g21} = -F_{g12}$ (N), N =Newtons,

G is the gravitational constant= $6.673 \times 10^{(-11)} N \frac{m^2}{kg^2}$

Note: Boldface typeset in equations indicates vector, and non-

mal typeset are scalars

Electric field (E) are produced by both moving charges and stationary charges. The electric field is represented as a vector that associates each point in space the Columbus forces as stated in eq. 2 and shown in Figure 3B), where the vectors are meaning from a positive electric charge to a negative.

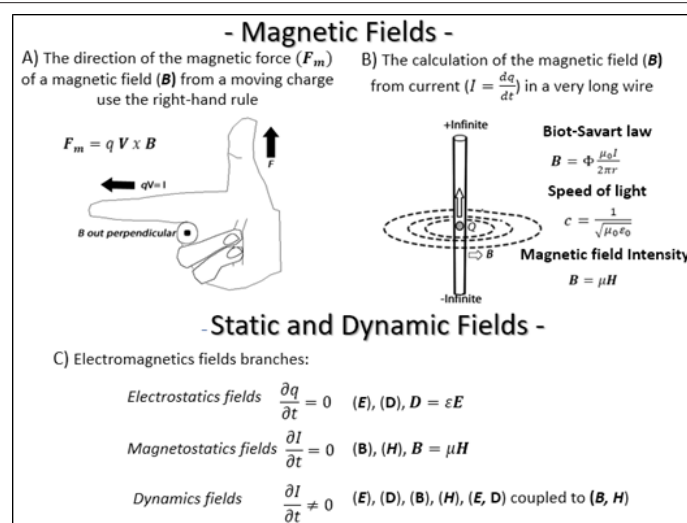


Figure 3: Magnetic fields: Statics and dynamics

A) Direction of magnetic force (), B) Calculation of magnetic field (B) in a long wire and C) Electromagnetic fields branches.

Eq. 2) Coulomb's law in for electric force in free space

$$\mathbf{F}_{e21} = \mathbf{R}_{12} \frac{q_1 q_2}{4\pi\epsilon_0 R_{12}^2} \text{ (N)} \quad N = \text{Newtons}$$

where: charge of single electron $q = -1.6 \times 10^{-19}$ (C), C=Coulomb

Electrical permeativity free space $\epsilon_0 = 8.854 \times 10^{-12} = \frac{1}{36} \pi 10^{-9} \left(\frac{F}{m} \right)$, $\left(\frac{F}{m} \right)$ = Faraday per meter

We can define the Electric Field Intensity as the strength of an electric field at any point, and it is equal to the electric force per unit charge experienced by a test at that point, stated in equation 3.

Eq. 3) Electric Field Intensity $\mathbf{E} = \mathbf{R} \frac{q}{4\pi\epsilon R^2}$ (V/m), V/m=volts per meter

where: R is the distance between the charge and the observation,

$\epsilon = \epsilon_r \epsilon_0 \left(\frac{F}{m} \right)$ where: relative permittivity or dielectric constant ϵ_r ($\epsilon_r = 1$ for vacuum)

For analysis, the *Electric Flux Intensity (D)* can be defined as the measure of the intensity of the electric field (*E*) generated by a free electric charge, corresponding to the number of electric field lines through a given area, as stated in equation 4.

Eq. 4) Electric Flux Intensity $\mathbf{D} = \epsilon \mathbf{E} \left(\frac{C}{m^2} \right)$, $\frac{C}{m^2}$ = Coulomb per squared meter**Magnetic Fields Equations Review**

The main difference between electric charges and magnetic poles are: Electric charges can be isolated, but magnetic poles always exist in pairs.

Magnetic flux density or magnetic induction (*B*) are the mag-

netic lines surrounding the magnetic poles; similar to electric fields (*E*), where they are produced only by moving charges. *B* can be represented in field lines that always close on themselves, which explains why magnetic field across a closed surface vanishes, a magnet with the magnetic flux (*B*) around its poles is shown in Figure 2.

A charged particle moving without acceleration produce an electric field (*E*) and a magnetic field (*B*). It produces the electric field because it's a charge particle, but when it is at rest, it doesn't produce a magnetic field. All of a sudden when it starts moving, it starts producing a magnetic field. To find the direction of the magnetic field density (*B*) use the right-hand rule: point thumb in direction of current (charge displacement) and the fingers will curl in the direction of the magnetic field as shown in Figure 2D (Figure 2).

Magnetic fields (*B*) create a magnetic force (*F_m*) only on moving charges (*q*). The vector from: Magnetic Force (*F_m*) magnetic field (*B*), and velocity of moving charge (*v*) are all perpendicular of each other. To find the direction of the magnetic force (*F_m*) of a magnetic field (*B*) from a moving charge use the right-hand rule as indicated in Figure 3 A) following the next steps:

- Pointer finger in the direction the positive charge is moving (*qV*), and
- then your middle finger in the direction of the magnetic field (*B*),
- your thumb points in the direction of the magnetic force (*F_m*), from the moving charge (*qV*).

The Biot-Savart law describe the magnetic field (*B*) generated by an electric current (*I*) in a very long wire in free space as stated in eq. 5 and shown in Figure 3B.

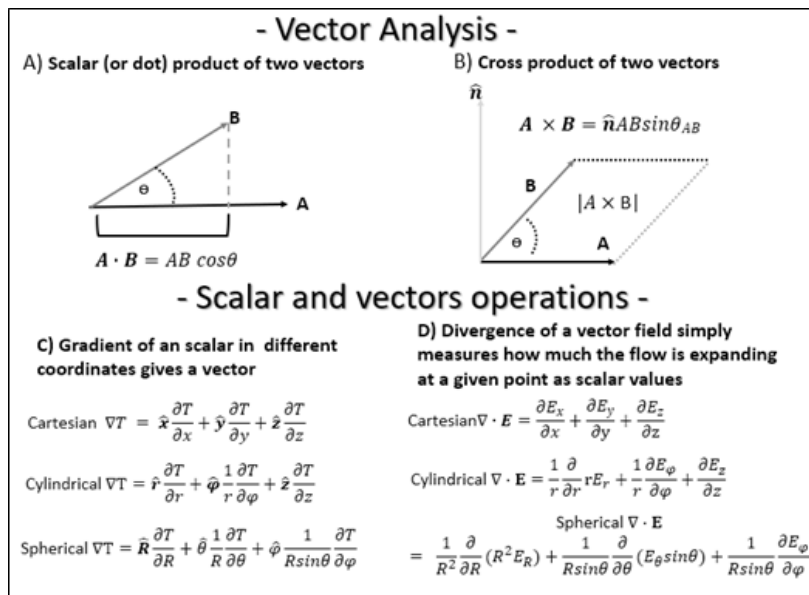


Figure 4: Vector Analysis

A) Scalar (or dot) product of two vectors, B) Cross product of two vectors, C) Gradient of a scalar, and D) Divergence of a vector field

Eq. 5) Biot-Savart law for magnetic field B in a long wire

$$\mathbf{B} = \Phi \frac{\mu_0 I}{2\pi r} \text{ (T)}, \text{ T=Tesla}$$

Where: Φ is an azimuthal unit that indicates the magnetic field direction

μ_0 is the magnetic permeability in free space $\mu_0 = 4\pi \times 10^{-7} \left(\frac{H}{m}\right)$, $\frac{H}{m}$ =Henry per meter

I =current (A) Amperes, r = radial distance from the current in (m) meters

The speed of light in vacuum (c) it is a universal physical constant which value is 299,792,458 meters per second (approximately 3.00×10^8 m/s), and it can be stated with respect to the magnetic permeability in free space (μ_0) and electrical permittivity free space (ϵ_0) by eq. 6 as follow:

Eq. 6) Speed of light in vacuum $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \left(\frac{m}{s}\right)$, $\frac{m}{s}$ =meters per seconds

The Magnetic field Intensity (H) that is related with magnetic flux density multiplied by magnetic permeability (μ) as stated in eq. 7

Eq. 7) Magnetic field Intensity $\mathbf{B} = \mu \mathbf{H} \left(\frac{A}{m}\right)$, $\frac{A}{m}$ =Ampere per meter

Where the magnetic permeability $\mu = \mu_r \mu_0 \left(\frac{H}{m}\right)$, $\frac{H}{m}$ =Henries per meter

Static and Dynamic Fields

To study electromagnetics fields are divided in three branches as shown in Figure 3C: Electrostatics, Magnetostatics and Dynamics. Where:

- Electrostatics fields focus on stationary charges (q). These means when $\frac{\partial q}{\partial t} = 0$, where the moving charges are represent-

ed by dc current. i.e. Electric field intensity (E), Electric flux density (D) and their relation $\mathbf{D} = \epsilon \mathbf{E}$

- Magnetostatics fields focus on stationary currents (I). These means $\frac{\partial I}{\partial t} = 0$, where the magnetic field does not depend on q , but rather on the rate of change (current) flowing through it. i.e. Magnetic flux density (B), Magnetic field intensity (H) and their relation $\mathbf{B} = \mu \mathbf{H}$
- Dynamics fields focus in time-varying currents (I). These means $\frac{\partial I}{\partial t} \neq 0$, where depends on currents and associated charge density. Dynamics fields involve a time-varying electric field that will generate a time-varying field and vice versa. i.e. Electric field intensity (E), Electric flux density (B), Magnetic flux density (B), Magnetic field intensity (H) (Figure 3).

Maxwell Equations

The theory of electromagnetism is based on a set of four fundamentals relations known as Maxwell's equations, The Maxwell equations were based from experimental observations reported by: Coulomb, Gauss, Ampere, Faraday and others to define three cases:

- Connection between the electric field E and electric charge q through the resulting field.
- Connection between the magnetic field intensity H and electric current I through current density J , and
- Describes the bilateral coupling between electric fields E and magnetic fields B and fluxes

The Maxwell equations are also known as: Gauss's law for electricity, Faraday's law, Gauss's law for magnetism and the Ampere's law are shown in (Table 1).

Table 1: The four Maxwell's equations.

The four Maxwell's equations divided by three electromagnetic branches: Electrostatics, Magnetostatics and dynamic fields.

Maxwell equations	Electrostatics fields	Magnetostatics fields	Dynamics Fields	
			Differential form	Integral form
Gauss's law for electricity	$\nabla \cdot \mathbf{D} = \rho_v$		$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$
Faraday's law	$\nabla \times \mathbf{E} = 0$		$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$ For stationary surface S
Gauss's law for magnetism		$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$
Ampere's law.		$\nabla \times \mathbf{H} = \mathbf{J}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int (\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}) \cdot d\mathbf{S}$

The use of the variables and parameters are as follow:

- The scalar (or dot) product *two vectors* as stated in the example of eq. 8 and shown in Figure 4A.

Eq. 8) Scalar (or dot) product of two vectors $\mathbf{A} \cdot \mathbf{B} = AB \cos \theta_{AB}$

\times is the vector (or cross) product of two vectors as stated in the

example of eq. 9 and shown in Figure 4B.

Eq. 9) Cross product of two vectors $\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}} AB \sin \theta_{AB}$ where $\hat{\mathbf{n}}$ is a unit vector to the plane AB

The gradient is a multi-variable generalization of the derivative. When a derivative can be defined on function of a single variable that gives as result a scalar value, the gradient is used for the derivative of functions with several variables that give as result a vector as indicated in eq. 10. The gradient can be used in different coordinates: cartesian, cylindrical and spherical, as shown in Figure 4C.

Eq. 10) ∇ is the gradient of a scalar, i.e. $\nabla T = \text{grad } T = \hat{x} \frac{\partial T}{\partial x} + \hat{y} \frac{\partial T}{\partial y} + \hat{z} \frac{\partial T}{\partial z}$
The divergence of a vector field simply measures how much the flow is expanding at a given point. It does not indicate in which direction the expansion is occurring; the divergence gives as a result a scalar value as indicated in eq. 11. The divergence can be used in different coordinates: cartesian, cylindrical and spherical, as shown in Figure 4D.

Eq. 11) $\nabla \cdot$ is the divergence of a vector, i.e. $\nabla \cdot \mathbf{E} = \text{div } \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$

(Figure 4)

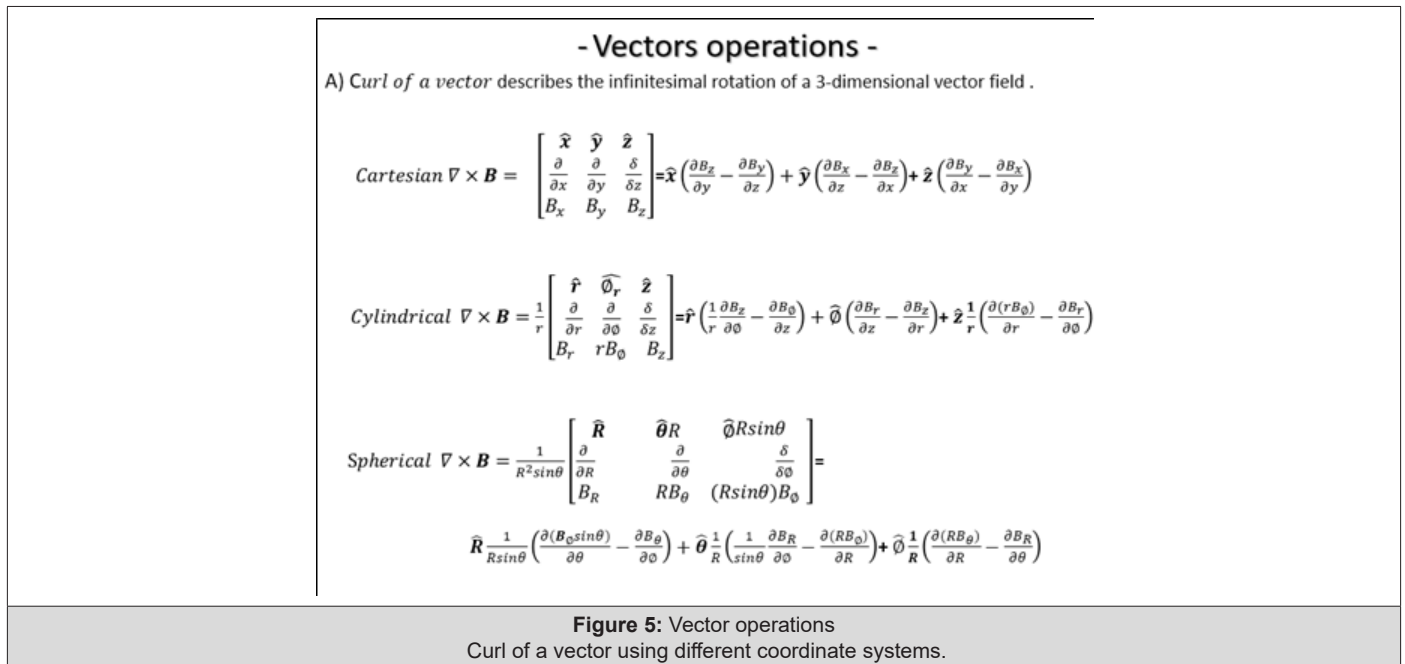
curl \mathbf{B} describes the infinitesimal rotation of a 3-dimensional vector field \mathbf{B} . At every point in the field, the curl of that point is represented by a vector characterizing the rotation at that point as indicated in eq. 12. The curl of a vector can be used in different coordinates: cartesian, cylindrical and spherical, as shown in Figure 5A.

Eq. 12) $\nabla \times$ is the curl of a vector $\nabla \times \mathbf{B} = \text{curl } \mathbf{B} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{bmatrix}$

$$= \hat{x} \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) + \hat{y} \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right) + \hat{z} \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right)$$

(Figure 5) The divergence theorem is extensively use in electromagnetics, it relates the flow of a vector field through a surface to the behavior of the vector field inside a surface, as stated in eq. 13 and shown in Figure 6A.

Eq. 13) Divergence theorem $\int_V \nabla \cdot \mathbf{E} \, dv = \oint_S \mathbf{E} \cdot d\mathbf{s}$



The *Stroke's theorem* converts the surface integral of the curl of a vector over an open surface S into a line integral of the vector along the contour C bounding the surface S, this simplified the calculus of many vector calculus as stated in equation 14.

Eq. 14) Stroke's theorem $\int_V (\nabla \times \mathbf{B}) \cdot d\mathbf{s} = \oint_C \mathbf{B} \cdot d\mathbf{l}$

The Laplacian operator find the combination of the divergence of a gradient of a scalar as stated in eq. 15) and in others coordinate at Figure 5B.

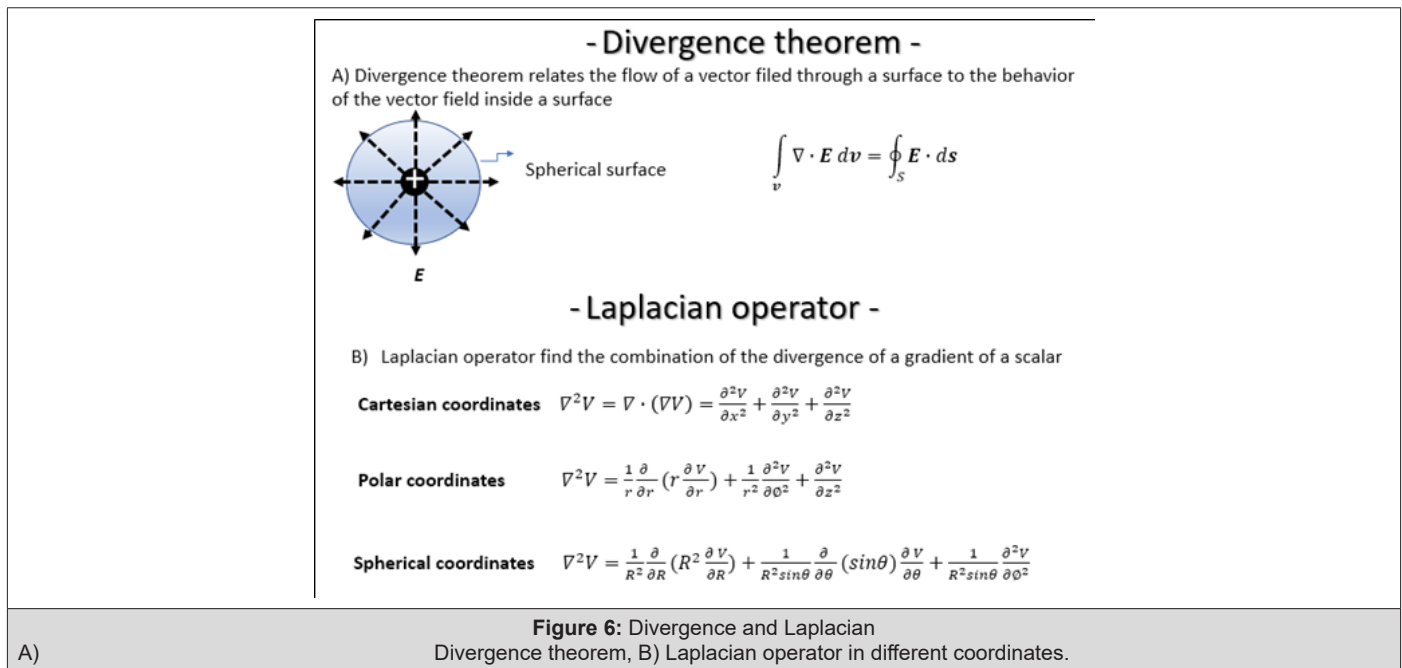
Eq. 15) Laplacian operator in cartesian coordinates

$$\nabla^2 V = \nabla \cdot (\nabla V) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

(Figure 6) ρ_v is the electric charge density per unit volume. It describes how much electric charge is accumulated in a particular field. Specifically, it finds the charge density per unit volume, but also can be used in surface area, and length.

\mathbf{J} is the current density or amount of electric current flowing through a unit cross-sectional area per unit area, we can get the current as indicated in equation 16.

Eq. 16) Current I from current density $I = \int_S \mathbf{J} \cdot d\mathbf{s}$ (A), A=Amperes



Electromagnetic Field Toroidal Application

Electromagnetism is used to describe one of the fundamental forces of nature serves as a basic principle of working for many natural effects and human developed devices; form the basic force found between subatomic particles such as protons and electrons that help to hold matter together to sophisticated electromagnetic Devices based in how a magnetic field is created by the flowing of electric current through the effect of electromagnetic waves that differ in wavelength as: radio waves, microwaves, visible light , x-rays. Etc. One of the most common and important electromagnetic devices is the Electromagnetic Field Toroidal and its applications like the sophisticated Thermonuclear stabilization using confined systems as explained in the next sections.

Calculation of the Toroidal Electromagnetic Magnetic Field

An Electromagnetic Field Toroidal or toroidal coil is built wrapping in closely space turns of wire around of a doughnut shape structure as shown in Figure 7A. Electromagnetic Field Toroidal

is a passive electronic component when a current I is applied to the coil all its wire loops contribute to the magnetic field B in the same direction inside the toroid, which direction is described by the right-hand rule indicated in Figure 2 D. This magnetic field B , present around all the toroidal core creates its own north and south pole, and the most of the magnetic field B is concentrated in its axial core, this is the reason of why this configuration is also known as *Toroidal Confined Electromagnetic Fields (TCEF)*.

As explain in eq. 5) the *Biot-Savart law* allows the calculation for magnetic field B in a long wire as $\mathbf{B} = \Phi \frac{\mu_0 I}{2\pi r}$, if we need to calculate the magnetic field B in only one loop of the toroid as shown in Figure 7B, when the current I is applied in one direction, create a magnetic field that it is concentrated in the central axis. For the calculation of B the variables indicated in Figure 7C are used, where: R is the radius of the loop, \hat{r} is the distance analyzed, I is the current applied, and $d\mathbf{L}$ is the incremental length of the loop analyzed. $d\mathbf{B}$ is the incremental magnetic field calculated at the center of the loop that can be obtained using the following equation based on the Ampere' law that which relate magnetic field and current in a general way:

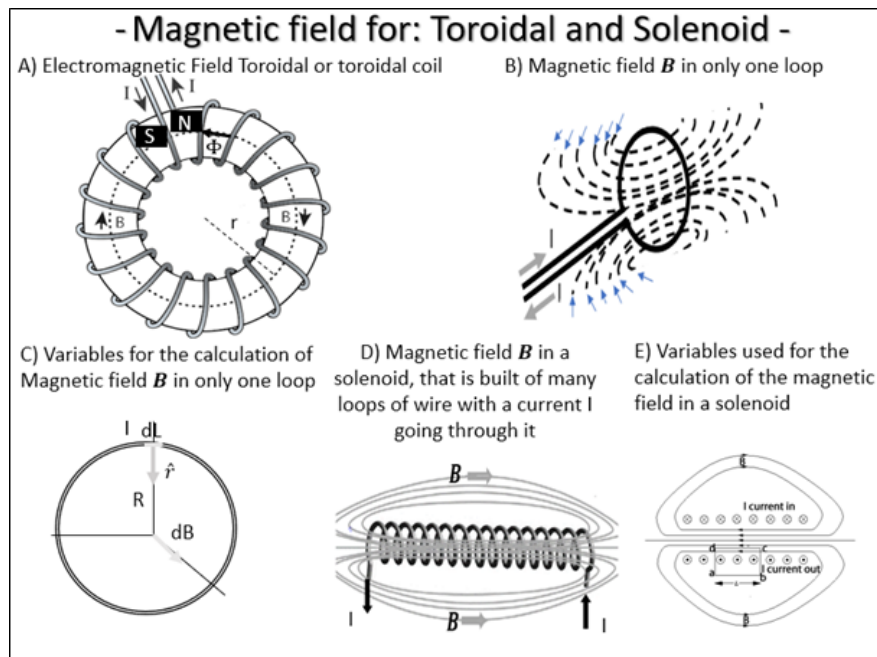


Figure 7: Magnetic field for Toroidal and Solenoid
Calculation of the magnetic field B for: Toroidal and Solenoid.

Eq.17 Magnetic field B in only one loop defined by Ampere's law

$$d\mathbf{B} = \frac{\mu_0 I d\mathbf{L} \times \hat{\mathbf{r}}}{4\pi R^2} = \frac{\mu_0 I dL \sin \theta}{4\pi R^2}$$

The equation 17 can be further simplified because $\theta = 90^\circ$ for all points around the loop as indicated in the following integration equation:

Eq.18 Magnetic field B in only one loop and its center is

$$\mathbf{B} = \frac{\mu_0 I}{4\pi R^2} \oint_c d\mathbf{L} = \frac{\mu_0 I}{4\pi R^2} 2\pi R = \frac{\mu_0 I}{2R}$$
 (in T=teslas or newtons per meter per ampere)

Where the result of the integral to calculate the magnetic field $\mathbf{B} = \frac{\mu_0 I}{2R}$ is related to the circumference of the circle and the current of the loop related with μ_0 that represents the constant of permeability on the free space

We know that the *Gauss's law for the magnetism* states that the magnetic field B has divergence equal to zero (see Table 1 Maxwell's laws), in other words, the basic entity for magnetism is the magnetic dipole as represented in the next equation:

Eq. 19) Gauss's law for the magnetism $\oint_c \mathbf{B} \cdot d\mathbf{S} = 0$

The Gauss's law for the magnetism can be proved in the solenoid, that is built of many loops of wire with a current I going through it, as shown in Figure 7D. Where, solenoids are used to generate magnetic field concentrated in one axis in the centre of the core and it is directed along the axis of the solenoid, but outside of the solenoid the magnetic field is far weaker. The winding is sufficiently tight that each turn of the solenoid is well approximated as a circular wire loop, lying in the plane perpendicular to the axis of the solenoid. Suppose that there are n turns per unit axial length of the solenoid and we want to calculate the magnetic field around the rectangle indicated as a, b, c, d in Figure 7E). We notice

that along the sides of the rectangles indicated as $b-c$ and $d-a$, the magnetic field is essentially perpendicular to the loop, so there is not contribution to the calculation to the line integral from these sections of the loop. Along $c-d$ the magnetic field is approximately uniform on magnitude, and is directed parallel to the loop. Thus, the contribution to the line integral from this section of the loop is, B and L , where L is the length of $c-d$ because along $a-b$ the magnetic field-strength is essentially negligible, so this section of the loop makes no contribution to the line integral. It follows that the line integral of the magnetic field around a, b, c, d is simplified to the next equation:

Eq. 20) Line integral of the magnetic field around a rectangle in a solenoid $\oint_c \mathbf{B} \cdot d\mathbf{S} = BL$

The *Ampere's law* states that for any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop, this was confirmed for the calculation of the magnetic field loop in equation 18), beside in equation 20) the line integral is equal to μ_0 times the algebraic sum of the currents which flow through the loop a, b, c, d . Since the length of the loop along the axis of the solenoid is L , the loop intersects n turns of the solenoid, each carrying a current I . Thus, the total current which flows through the loop is nLI . This current counts as a positive current since if we look against the direction of the currents flowing in each turn, the loop a, b, c, d circulates these currents in an anti-clockwise direction. Ampère's circuital law yields to the equation:

Eq. 21) Long Solenoid magnetic field $\mathbf{B}L = \mu_0 nLI$ which can be reduce to $\mathbf{B} = \mu_0 nI = \frac{\mu_0 NI}{l}$ for $\frac{L}{R} \gg 1$,

where, N the numbers of turns* and n is the number of turns per unit length in a solenoid related as $n = \frac{N}{l}$

Note: * The turns are tightly wound implies that the pitch of a single turn is small compared with the solenoid's turns.

In summary, the magnetic field in the core of a solenoid $B = \frac{\mu_0 n I}{l}$ is directly proportional to the product of the current flowing around the solenoid and the number of turns per unit length in it. This result is exact in the limit in which the length of the solenoid is very much greater than its diameter [1].

Because toroid is a closed loop most magnetic field is kept in its core, and all the loops of the coil on the toroidal contribute to the total magnetic field B in the same direction inside the toroid indicated by the already explained right hand rule in Figure 2D. The formula for a toroid approximates that for the long solenoid, the only difference being that L is now instead given by the circumference of a toroidal axis equal to $2\pi r$ as indicated in the next equation:

$$\text{Eq. 22) Magnetic field of a toroid } B_t = \frac{\mu_0 n I}{2\pi r} \left(\frac{W_b}{m^2} \right)$$

where: μ_0 is the permeability of free space, n is the number of turn around the toroid,

I is the current in each turn, and r the radius of the toroid as indicated in Figure 7A (Figure 7).

The solenoid and the toroidal are known as inductors or coils that are built of multiple turns of wire helically coiled around a cylindrical core that may contain air filled or a magnetic material with permeability μ . The self-inductance or inductance of a solenoid is the ratio of the current and the flux linkages that the current produces, as described in the next equation:

$$\text{Eq. 23) Inductance of a solenoid } L = \frac{\Lambda}{I} \text{ (in Henry=Webbers/Amp)}$$

Where: Λ is the flux linkage for a solenoid, and I is the current.

The total magnetic flux flowing through the solenoid can be found with the integration across the cross-section of the solenoid as:

$$\text{Eq. 24) Magnetic flux in a solenoid } \Phi = \iint_S \mathbf{B} \cdot d\mathbf{S} = \frac{\mu N I}{l} S \text{ (in } W_b = \text{Webber)}$$

Where: S is the cross-sectional area of the solenoid that can be calculated as $S = \pi r^2$, when the solenoid is circular with the radius r .

The total flux linkage is just the product of the magnetic flux by the number of loops as indicated in equation 25.

$$\text{Eq. 25) Total flux linkage in a solenoid } \Lambda = N\Phi = \frac{\mu N^2 S}{l} I$$

And combine this last equation with eq. 23 to obtain in other terms the inductance of a solenoid:

$$\text{Eq. 26) Inductance of a solenoid } L = \frac{\Lambda}{I} = \frac{\mu N^2 S}{l} \text{ (in Henry } H = \frac{W_b}{A})$$

Where, μ_0 is the magnetic permeability in free space ($4\pi \times 10^{-7}$), N is the numbers of turns in the coil, S is the inner core area of the solenoid (πr^2 in m^2) and l the length of the coil in m .

For analogy we can get the inductance of a toroidal as:

$$\text{Eq. 27) Inductance of a toroidal } L = \mu \frac{N^2}{2\pi r} S \text{ (in Henry } H = \frac{W_b}{A})$$

where S is the cross-sectional area of the wire, r the radius of the toroidal and μ is the magnetic permeability.

The solenoid and the toroidal have the capacity of store energy in the magnetic field near its current-carrying conductors. This energy is described for the equation 28:

$$\text{Eq. 28) Inductance magnetic energy } W_m = \frac{1}{2} L I^2 \text{ (in Jules J)}$$

This means because the current is elevated to square to calculate the inductance magnetic energy this will allow us to store a reasonable amount of energy W applying a small current I .

Toroidal Application: Transformer

A transformer is a static electrical device that transfers electrical energy between two or more circuits through the principle of electromagnetic induction. A varying current in one coil of the transformer produces a varying magnetic field, which in turn induces a voltage in a second coil. A varying current in one coil of the transformer produces a varying magnetic field, which in turn induces a varying electromotive force (emf) or voltage in a second coil. Power can be transferred between the two coils, without a metallic connection between the two circuits. *Faraday's law of induction* discovered in 1831 described this effect. Transformers are used to increase or decrease the alternating voltages in electric power applications. For many years, closed-core inductors and transformers often used cores with a square shape as shown in Figure 8A, the use of toroidal-shaped cores as a transformer has increased greatly because of their superior electrical performance, see Figure 8B. The main advantage of the toroidal shape is that, due to its symmetry, the amount of magnetic flux that escapes outside the core (leakage flux) is low, therefore it is more efficient and thus radiates less *Electromagnetic Interference (EMI)*.

Mutual inductance is the magnetic coupling between two conducting structures. Mutual inductance allows to build transformers where the loops share a common magnetic core. If we have two multiloop closed loops with surface S_1 with N_1 turns named primary and S_2 with N_2 turns named secondary. The current I_1 flows through the primary creating a of the magnetic field B and no current flows in the secondary because in this example is an open circuit, then the magnetic flux Φ_{12} as shown in Figure 8B can be calculated as indicated in eq. 29 [2]:

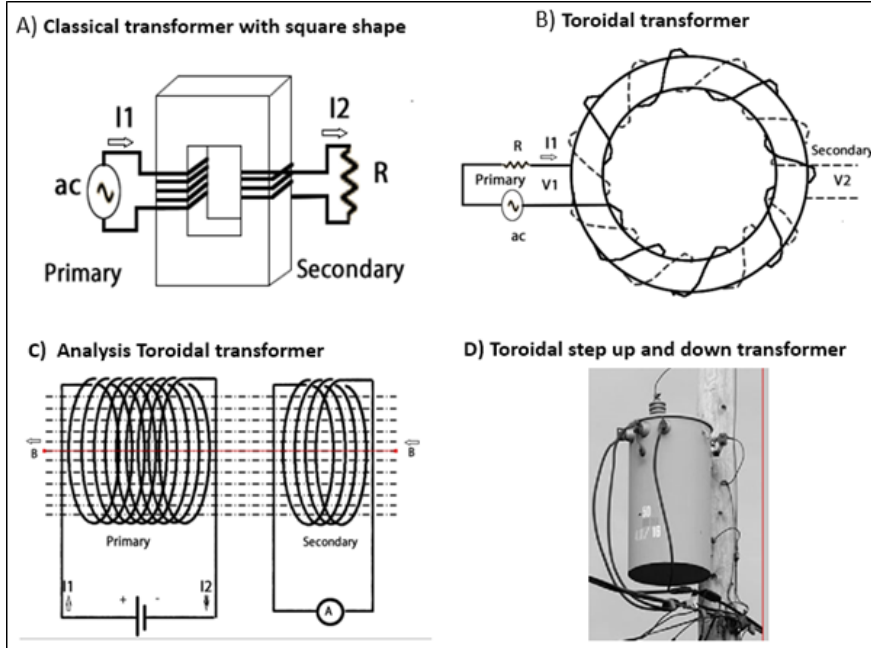


Figure 8: Transformers: Square and Toroidal core.

Eq.29 Magnetic flux from primary to secondary coil

$$\Phi_{12} = \int_{S_2} \mathbf{B}_1 \cdot d\mathbf{S} \text{ Wb (Weber)}$$

And the mutual inductance can be obtained as:

Eq. 30) Mutual inductance in a transformer $L_{12} = \frac{N_2}{I_1} \int_{S_2} \mathbf{B}_1 \cdot d\mathbf{S}$ (H=Henry)

The magnetic energy is obtained from the power p that is equal to the voltage v multiplied by the current i as follow:

Eq. 31) Magnetic energy in a transformer $w_m = \int p \, dt = \int v i \, dt = L \int_0^I i \, di = \frac{1}{2} L I^2$ (J=joules)

When a galvanometer (predecessor of the voltmeter) detects the flow of current through the coil, shows that a voltage called emf (electromotive force) as has been induced (electromagnetic induction) as indicated in the next equation attributed to Faraday's law and as shown in Figure 8C.

Eq.32 Voltage emf in a transformer $V_{emf} = -N \frac{d\Phi}{dt} = -N \frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S}$ (V=volts)

In transmission lines AC power is much more efficient and easier to make thanks through transformers ability to produce voltage V with change in magnetic flux. AC current, which travels as sinusoidal waves, produce AC V produced by the changing flux. On the primary the voltage source V_1 generates current I_1 that establish the magnetic flux Φ , and from this a similar relation can be obtain in the secondary

Eq. 33) Transformer relation voltages primary $V_1 = -N_1 \frac{d\Phi}{dt}$ **Eq. 34) Transformer relation voltages secondary** $V_2 = -N_2 \frac{d\Phi}{dt}$

and combined eq. 33 and 34 them

Eq. 35) Transformer relation voltages between primary and secondary $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

In an ideal transformer the core has an infinite permeability ($\mu = \infty$), and the magnetic flux is confined within the core, then no power is lost and the powers in the primary and secondary are equal.

Eq 36) Ideal Transformer $P_1 = P_2$ then $V_1 I_1 = V_2 I_2$

and combining equation 33 with equation 35 we can obtain:

Eq. 37) Transformer relation currents between primary and secondary $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

The electric power based on AC delivery systems is based on toroidal step up and down transformers using different number of coils to get voltage needed as 400 Kv, 220 v, 110 v as shown in Figure 8D (Figure 8).

Thermonuclear Stabilization Using Confined Systems

Thermonuclear fusion is a way to achieve nuclear fusion by using extremely high temperatures. There are two forms of thermonuclear fusion: uncontrolled and controlled, where [3]:

- Uncontrolled nuclear fusion is when the resulting energy is released in an uncontrolled manner, as it is in thermonuclear weapons and in most stars.
- Controlled nuclear fusion is when the fusion reactions take place in an environment allowing some or all of the energy released to be harnessed for constructive purposes.

In the nuclear fusion at higher temperature of the material (usually plasma*), we will gain energy after reaching sufficient temperature, the energy obtained of from collisions within the plasma is high enough and the particles may fuse together. This behavior is defined by the electrostatic potential energy equation defined by the Coulomb barrier as shown in equation 36.

Eq. 38) Electrostatic potential energy $Barrier_{Coulomb} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$ (J) Joules

Where: ϵ_0 is the permittivity of free space $\approx 8.854 \times 10^{-12} \frac{F}{m}$ Farads per meter

q_1, q_2 are the electrical charges of the particles involved

r is the radius between the the particles involved

Note: * Plasma is one of the four fundamental states of the matter that are: solid, liquid, gas and plasma. It can be artificially generated by heating a neutral gas; the plasma is electrically conductive, produce magnetic fields and electric current and respond strongly to electromagnetic forces.

The temperature is a measure of the average kinetic energy of particles involves in the fusion, usually this temperature is when the energy is equally or greater than 0.1 MeV (Mega Electron Volts), obtained as shown in equation 37.

Eq. 39) Electron volt $E = qV(eV)$ Electron Volt where $1 eV \approx 1.602 \times 10^{-19}$ (J) Joules

Making the conversion of energy .1 MeV to temperature gives 1.2 billons kelvins, using the conversion of thermal energy in the internal energy of an object due to the kinetic energy of its atoms; this is shown in equation 38.

Eq. 40) Thermal energy $Q = mc\Delta T$ (J) joules,

where: m is the mass of substance, c is specific heat of substance and

ΔT change of temperature obtained from initial minus final temperature in Celsius

The main problem is how the confined the hot plasma of 1.2 billons kelvins or higher temperature. This is so hot that cannot be in direct contact with any solid material, so it has to be located in the vacuum; besides that, high temperature creates high pressure. The plasma trends to expand and some force is needed to maintained in place, for this there is three methods for the confinement of the energy created: Gravitational, Magnetic, inertial and Electrostatic. Where each one is explained in the next sections:

Confined Gravitational System

Confined gravitational field is a force based on gravity generated from the size of a big mass, this requires a stable equilibrium defined by a local minimum of the gravitational potential in the middle of empty space as shown in eq. 41. This equation can be obtained defining the gravitational field (g) as the gradient of a scalar that has a zero curl as indicated in eq. 41, and then calculat-

ing the differential form of Gauss 'law for gravity that becomes the Poisson's equation for gravity shown in equation 40.

Eq. 41) Gravitational field $g = -\nabla\phi$

Eq. 42) Poisson's equation for gravity $\nabla^2\phi = 4\pi G_p$

In other words, in radial symmetric mass the gravitational potential is a function of the radius ($r=|r|$) and the Poisson equation becomes the eq.43 when we apply the Laplacian operator indicated in Figure 6B).

Eq. 43) Poisson's equation for gravity based on radius

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \phi}{\partial r} \right) = 4\pi G_p$$

To achieve the equilibrium of the gravitational field to contain the nuclear fusion and its extremely high heat, the next equation must be equal to zero.

Eq. 44) Stable equilibrium of the gravitational field $\nabla^2\phi = 4\pi G_p = 0$ in an empty region of space

Conclusions on confined gravitational field:

A confined gravitational field can only be achieved with another big mass as stars or planets to obtain a stable equilibrium needed to contains the nuclear fusion and its high temperature.

Inertial Confinement System

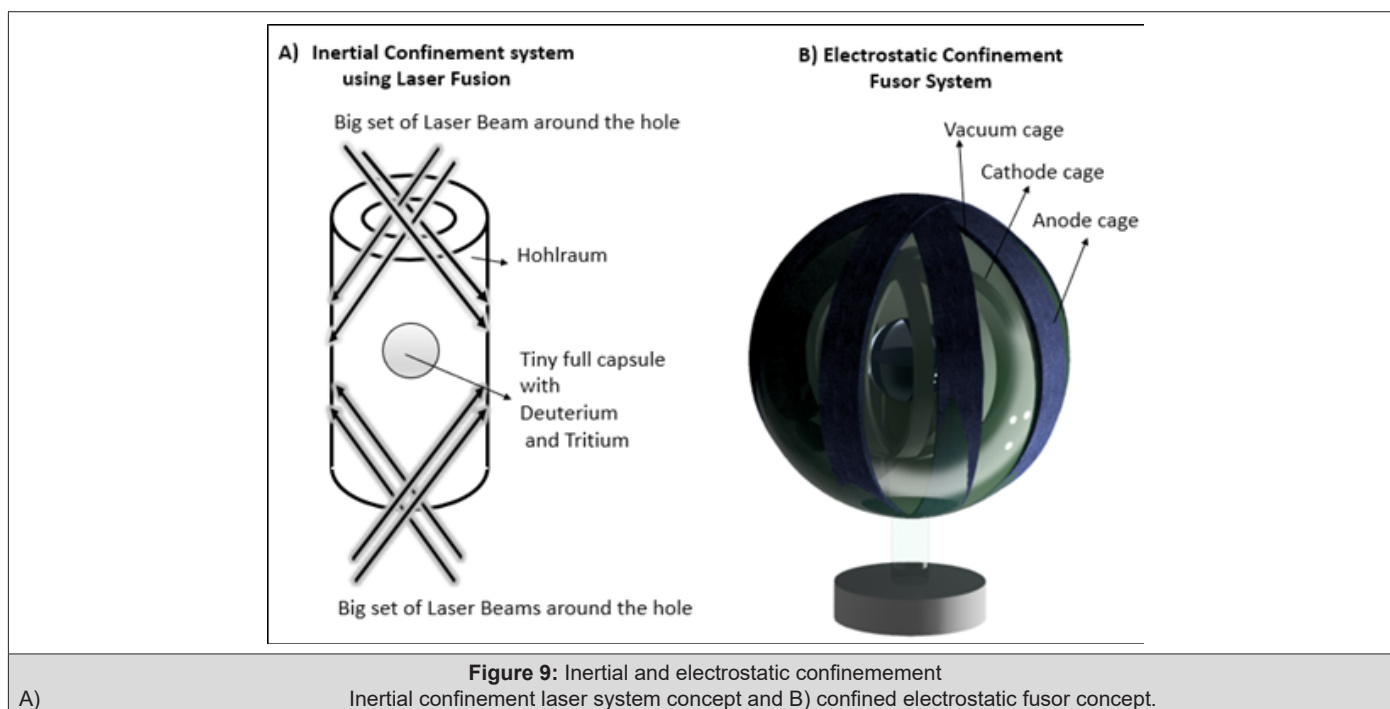
Inertial confinement system is when fuse nuclei so fast that they don't have time to move apart. This is a type of fusion energy research that attempts to initiate nuclear fusion reactions by heating and compressing a fuel target of fuel with electrons (where electric charge is negative) or ion (where a non-zero net electric charge is present), typically in the form of a pellet that most often contains a mixture of *deuterium* (hydrogen with one neutron) and *tritium* (hydrogen with two neutrons) where the two are heavy isotopes of *hydrogen*.

The more frequently approaches to inertial confinement have been laser fusion. In the inertial Confined system using laser fusion, the goal is compress and heat the fuel, to obtain energy that is delivered to the outer layer of the target using a big set of high-energy beams of laser light [4]. A laser fusion attempts to force nuclear fusion in tiny pellets (micro-balloons of a *deuterium-tritium mixture*) by zapping them with such a high energy density that they will fuse before they have time to move away from each other as shown in Figure 9A [5], a group of the lasers beams are directed inside of an small gold cylinder known as *hohlraum*, where is a tiny full capsule that contains atoms of *deuterium and tritium* that fuel the ignition process. Once that the fusion ignition is created, the energy generated from the reaction outstrips the rate at which the laser-produced x-ray radiation gas forming a surrounding plasma envelope and electron conduction cool the implosion but generated a big compressing inward of the nucleous unit it reaches the temperature and density where fusion begin. This is created only in a tiny fraction of a second and it will produce ten to 100 times more energy than it was used to ignite it.

Conclusions on Inertial confinement system using laser fusion.

This method create energy from 1 to 100 time that the one used for its ignition but only for a tiny fraction of sec. The research on

this kind of confinement continue using more powerful laser fusion beams and ion-bean fusion with the goal of improve the method in the future (Figure 9).



Electrostatic Confinement System

Electrostatic confinement system are devices that confine ions (atom or molecule that has non-zero net electrical charge) using electrostatic fields. This device known as *fusor machine* has a cathode inside an anode wire cage in vacuum, where the positive ions fly towards the negative inner cage, if they miss, they can collide and fuse, besides they are heated by electrostatic field in the process. One general approach is shown in Figure 9B, where the machine makes a voltage between two metal cages, inside a vacuum. Positive ions fall down this voltage drop, building up speed. If they collide in its paths, they can fuse [6]. For every volt that an ion of ± 1 charge is accelerated across it gains 1 electron volt in energy, similar to heating a material by 11,604 kelvins in temperature. After being accelerated by 15 kV a singly-charged ion has a kinetic energy of 15 keV, similar to the average kinetic energy at a temperature of approximately 174 megakelvins, a typical magnetic confinement fusion plasma temperature. The common sources of the high voltage are ZVS Flyback HV source and Neon Sign Transformers; it can also be called a particle accelerator. The Fusors performed nuclear fusion of low performance using simple fusor machines. The confined electrostatic fusor system is the classic application for Electrostatic fields using two of the four Maxwell's equation: The Gauss's law and Faraday law as indicated in Table 1 of section 1.3 and in eq. 45 and eq. 46

Eq. 45) Gauss's law for Electrostatics fields $\nabla \cdot \mathbf{D} = \rho_v$

Eq. 46) Faraday's law for Electrostatics fields $\nabla \times \mathbf{E} = 0$

Conclusion on Electrostatic confinement system

Because most of the ions fall into the wires of the cage, fusors suffer from high conduction losses. On a bench top, these losses can be at least five orders of magnitude higher than the energy released from the fusion reaction, even when the fusor is in star model [7] Hence, no fusor has ever come close to break-even energy output, but the research continues using different configurations and methods.

Magnetic Confinement System

The magnetic confinement fusion is an approach to generate thermonuclear fusion power that uses magnetic fields to confine the hot fusion fuel in a form of a plasma. This is made by the electrically charged particles that will follow magnetic field lines [8,9]; then the fusion fuel at high temperature can be trapped using a strong magnetic field that is created inside of toroidal shapes as explained in section 2. It is my personal goal in this research paper focus in the use of magnetic confinement due to the central use of the toroidal and its amazing magnetic confinement fields as a method for thermonuclear encasement. Under the extreme amount of heat in the plasma the Magnetic Confinement fusion is a successful container to encase the tremendous amount of energy produced. A classic and more common approaches dates form 1940 and they are: Tokamaks, Stellarators, and Mirror Confinement systems. Where each is explained in the next sections

Tokamaks

As stated in The Great Soviet Encyclopedia, the Tokamak was first developed in the Kurchatov Institute of Moscow in the 1950's. This machine after much research and development was built in 1968 and proved successful as a means of creating a reaction and encasing the hot plasma produced by such, through creation of magnetic field walls. Nowadays, the Tokamak is one of the most common machines used for nuclear reaction research [10]. The Tokamak machine has an enclosed toroidal geometry, where a coil is around it with the purpose of generate a magnetic field of the toroidal (B_t) within the cavity, additionally has some other magnetic field in the core (B_c) generating by a coil (I_c). The fuel inside the cavity is made of *hydrogen isotopes* such as *deuterium and tritium*, the flow of this magnetic field creates the plasma circulation and it elevates the temperature within. Other forms de energy is also used to help the elevation of temperature to reach the fusion reaction needed, such as radio frequency heating and injection of high-energy particle beams. The Tokamaks main parts are shown in Figure 10 [11], these are: Toroidal frame, Toroidal field coil, Magnetic transformer cores and the Plasma

- Toroidal frame with radius R that has a metal conducting wall that is clad in its interior with graphite to help withstand the extreme generated heat of the plasma [12].
- Toroidal field coil that concentrate the magnetic field of the toroidal (B_t) in the axial center of it as explained in section 2 (Electromagnetic Field Toroidal app) of this paper and indicated in Figure 10B
- Magnetic transformer cores that generate an ohmic heating poloidal field (B_c), that help in the elevation of the temperature to reach the fusion reaction, this is indicated in Figure 10C
- Plasma is the fuel inside the toroidal cavity with the hydrogen isotopes: 2H (deuterium as a stable isotope of H -hydrogen that is the lightest element on the periodic table, that contains one proton and one neutron) and H (tritium as a radioactive isotope of hydrogen that contain 1 proton and 2 neutrons). This circulated plasma through the axial center is moved and heated for the resulting fields of ($B_t + B_c$) as indicated in Figure 10D, where the combination of both of these magnetic fields create a helically rotating resulting field that can successfully encase and compress the plasma, to handle the elevated temperatures of over 100-million-degree Kelvin, allowing the plasma to produce a nuclear reaction. Therefore, as most ex-

plosions tends to explode in an outwardly fashion; to prevent this outward explosion that could highly inefficient in terms of energy production as the plasma would quickly cool down and stop the fusion the resulting magnetic fields ($B_t + B_c$) are used to control producing a dynamic rotating magnetic field that compresses the plasma. Where the two dynamic magnetic fields relation from the Maxwell's equations are shown in in Table 1 of section 1.3 and identified as *Faraday's law for dynamic fields* as shown in eq. 45 and *Ampere's law for dynamic fields* as indicated in equation 46

Eq. 47) Faraday's law for dynamics fields $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

Eq. 48) Ampere's law for dynamics fields $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$

Conclusion on Tokamaks

The Tokamak is a successful Magnetic Confinement System that uses magnetic field to confine plasma in the shape of a torus. It Achieve a stable plasma equilibrium that requires magnetic field lines that move around the torus in a helical shape; such a helical field can be generated by adding a toroidal field B_t (traveling around the torus in circles) and a poloidal field B_c (traveling in circles orthogonal to the toroidal field). Research continues improving efficiency of this amazing confinement system.

Stellarators

Stellarators are confined magnetic fusion device that uses external magnets to confine the plasma and its extreme high temperature. It was invented by Lyman Spitzer of Princeton University in 1951, and much of its early development was carried out in the *Princeton Plasma Physics Laboratory (PPPL)* [13]. Essentially the Stellarator is build with two toroidal shapes of magnets that has been cut in half and then attached together with straight crossover sections forming a number 8 as indicated in Figure 11 A.

This design has the effect of propagating the nuclei from the inside to the outside as it orbits the device, thereby cancelling out the drift across the axis, at least if the nuclei orbit is fast enough. Not long after the construction of the earliest Figure-9 machines, it was noticed the same effect could be achieved in a completely circular arrangement by adding a second set of helically-wound magnets on either side. This arrangement generated a field that extended only part way into the plasma, which proved to have the significant advantage of adding shear, which suppressed turbulence in the plasma.

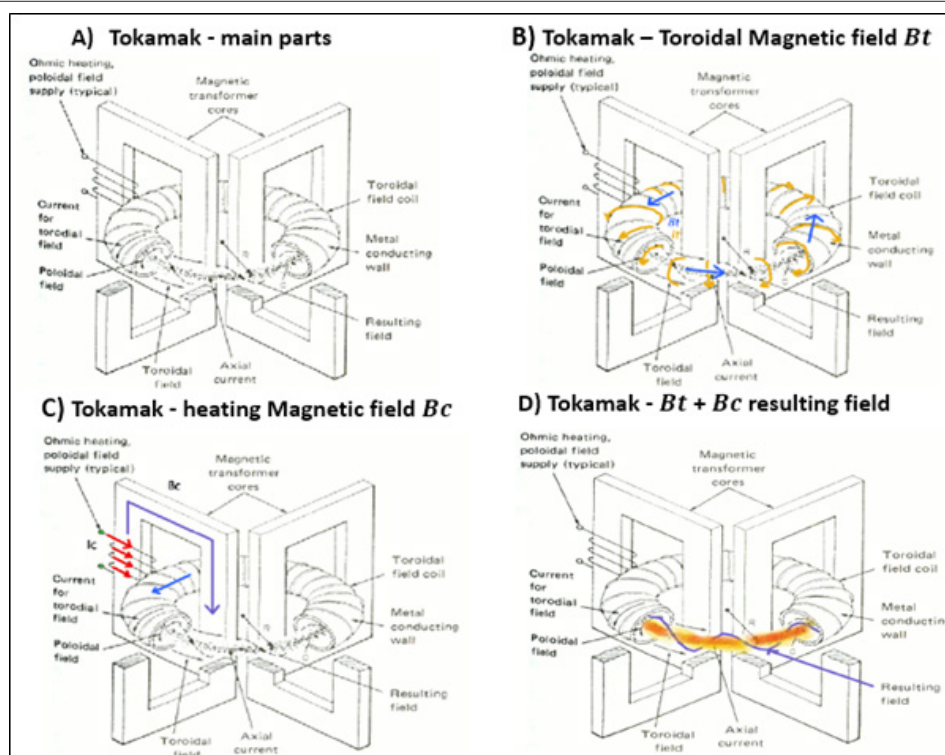


Figure 10: Tokamak [11].

A) main parts, B) Toroidal Magnetic field B_t , C) heating Magnetic field B_c and D) $B_t + B_c$ resulting dynamic rotating magnetic field.

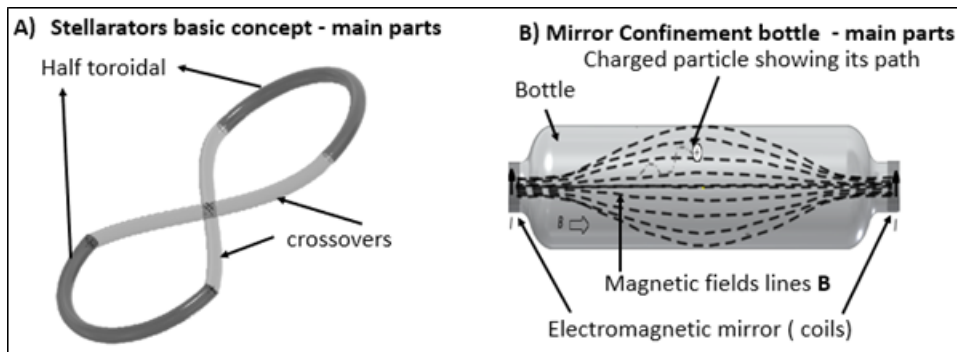


Figure 11: Stellarators and Mirror Confinement systems.

Two classic approaches: A) Stellarators, and B) Mirror Confinement systems.

Conclusion on Stellarators

When as larger Stellarators devices were built, it was seen that plasma was escaping from the system much more rapidly than expected, much more rapidly than could be replaced. By the mid-1960s. In addition to the fuel loss problems, it was also calculated that a power-producing machine based on this system would be enormous, the better part of a thousand feet long. Stellarators have seen renewed interest since the turn of the millennium, newer models have been built, but these remain about two generations behind the latest tokamak designs [14].

Mirror Confinement Systems

Mirror confinement systems known as a magnetic trap or pyrotron is a type of magnetic confinement technique used to trap the extreme high temperatures of the plasma using magnetic fields [15]. The main parts of a magnetic mirror confinement bottle are shown in Figure 11B, they are: electromagnets at both ends that generate a very density field (B), the charged particles (plasma) and the vacuum bottle. In a mirror confinement a configuration of electromagnets is used to create an area with a big density of magnetic field lines (B) at both ends of the confinement area. The particles approaching the ends receive an increased magnetic force

that cause them to reverse direction and return to the confinement area [16].

Conclusion on Mirror Confinement System

The basic mirror concept is inherently unstable, because the mirror effect occurs only in a limited range of velocity and angles. The device has been redesigned continuously under different as the: Tandem Mirror, minimum -B or ying-yang magnet and others that require even- large or more strong magnets. The research continues today in different countries like USA, Japan and Russia.

Conclusions

This paper introduces you to the applications of electromagnetic fields and electromagnetic methods of generate energy in the near future, and how they can be applied in the different thermonuclear stabilization confined systems. Where the Tokamaks devices has been very successful as a Magnetic Confinement system and Tokamaks are in continuously experimentation as the megaproject from the ITER (International Thermonuclear Experimental Reactor) in Saint-Paul-lès-Durance, in Provence, southern France. As an international fusion research that could be the world's largest magnetic confinement plasma physics experiment, funded by European Union, India, Japan, China, Russia, South Korea, and the United States. The ITER thermonuclear fusion reactor has been designed to produce a fusion plasma equivalent to 500 megawatts (MW) of thermal output power for around twenty minutes while 50 megawatts of thermal power are injected into the tokamak, resulting in a ten-fold gain of plasma heating power. Thereby the machine aims to demonstrate the principle of producing more thermal power from the fusion process than is used to heat the plasma, something that has not yet been achieved in any fusion reactor [17]. This kind of fusion reactor would produce virtually no CO₂ or atmospheric pollutants, and its radioactive waste products would mostly be very short-lived compared to those produced by conventional nuclear reactors.

We can conclude that different thermonuclear stabilization confined systems are scientifically and technological feasibility of fusion energy for peaceful use [18] as the full-scale electricity-producing fusion power stations and future commercial reactors [19].

Acknowledgement

None.

Conflict of Interest

Knowledge and best practices in this field are constantly changing as new research and experience broaden our understanding. Changes in research methods, professional practices, or physical treatment may be necessary. To the fullest extent of the law, neither the publisher nor the authors, contributors, or publishers assume any liability for any injury and/or damage to persons or property as a matter of product liability, negligence, or otherwise, or for any use or operation of any method, product, instruction, or idea contained herein.

References

1. Smith P (2010) A level physics notes: Electricity - self-inductance and mutual inductance. Retrieved from http://www.astarmathsandphysics.com/a_level_physics_notes/electricity/a_level_physics_notes_self_inductance_and_mutual_inductance.html.
2. Fawwaz T, Ulaby Umberto Ravaoli. Fundamentals of Applied Electromagnetics. Prentice Hall.
3. (2019) https://en.wikipedia.org/wiki/Thermonuclear_fusion.
4. (2019) <http://hyperphysics.phy-astr.gsu.edu/hbase/NucEne/finert.html>.
5. (2019) <https://lasers.llnl.gov/science/icf>.
6. (2019) <https://en.wikipedia.org/wiki/Fusor>.
7. Hedditch J (2015) Physics of Plasmas. Fusion in a Magnetically Shielded grid inertial electrostatic fusion device.
8. (2019) https://en.wikipedia.org/wiki/Magnetic_confinement_fusion.
9. (2019) https://en.wikipedia.org/wiki/Thermonuclear_fusion.
10. Garza-Ulloa J (2019) https://www.researchgate.net/publication/235929760_Electromagnetic_Field_Toroidal_applications.
11. Reference standard Handbook EE.
12. (2019) <https://en.wikipedia.org/wiki/Tokamak>.
13. (2019) <https://en.wikipedia.org/wiki/Stellarator>.
14. (2019) https://en.wikipedia.org/wiki/Magnetic_confinement_fusion#Stellarators.
15. (2019) https://en.wikipedia.org/wiki/Magnetic_mirror.
16. Fitzpatrick Richard (2011) Magnetic Mirrors. Home Page for Richard Fitzpatrick. The University of Texas at Austin.
17. (2019) <https://en.wikipedia.org/wiki/ITER>.
18. Meade Dale (2010) 50 years of fusion research. Nuclear Fusion. 50 (1): 014004. Bibcode:2010NucFu.50a4004M.
19. (2018) Broader Approach agreement. Iter.org. Retrieved.